ENGSCI 332
Control Systems

Lecture 5
Design of Feedback control

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Design Requirements

- Stability – don't want to lose control
  - So need decaying exponentials in response
  - But how quickly do they decay?
- Response speed – constrained by available power
- Accuracy – want minimum error once system converged
- Sensitivity to noise and disturbances

- Let controlled system transfer function be

$$G_C(s) = \frac{B_C(s)}{A_C(s)} = \frac{b_0 + b_1 s + \ldots b_m s^m}{a_0 + a_1 s + \ldots a_n s^n}$$

System stability

- By definition of Laplace transform, poles must be on LH plane for system to be stable (ie real part of roots <0)
- If we express transfer function as polynomial fraction, can write as:

$$\frac{Y(s)}{R(s)} = \frac{K(s-z_1)(s-z_2)...}{(s-p_1)(s-p_2)...}$$

  - i.e. as poles and zeros, with gain K.
  - In general, values of p_i and z_i will vary with K.
- Want all poles to be on left half plane for all values of K.
  - Root Locus – graphical representation of poles versus K.
  - But how far from axis is far enough?

Control system equations

- Controller: $C(s) = \frac{P(s)}{L(s)}$
- Plant (open-loop): $G_o(s) = \frac{B_o(s)}{A_o(s)}$
- Closed-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{C(s)G_o(s)}{1+C(s)G_o(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{P(s)B_o(s)}{L(s)A_o(s)+P(s)B_o(s)}$$

- In effect, have moved the open-loop poles $A_o(s)$ by adding coefficients from the numerators of plant and control functions
- How do we choose appropriate controller function *C(s)* to ensure that overall poles are on LHP (and yet meet design requirements)??

Accuracy

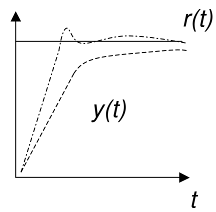
$$G_C(s)=\frac{B_C(s)}{A_C(s)}=\frac{b_0+b_1s+\ldots b_ms^m}{a_0+a_1s+\ldots a_ns^n}$$

- Steady-state accuracy
  - i.e. r(t) = α (for t≥0)

$$y_s(t)=\lim_{t\to\infty} sY(s)=\lim_{s\to 0} sG_c(s)\frac{\alpha}{s}=\alpha G_c(0)=\alpha\frac{b_0}{a_0}$$

  - Relative error

$$=\lim_{t\to\infty}\left|\frac{\alpha-y(t)}{\alpha}\right|=\left|\frac{\alpha-\alpha\frac{b_0}{a_0}}{\alpha}\right|=\left|\frac{a_0-b_0}{a_0}\right|=\left|1-G_c(0)\right|$$

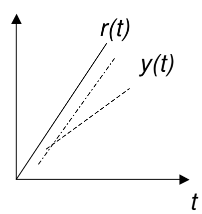


  - So, position error depends only on a_0 and b_0
  - If a_0 and b_0 equal, achieve zero position error

Accuracy

- Velocity error
  - ie $r(t) = \alpha t$ (for $t\geq 0$) - Ramp input, $R(s)=\frac{\alpha}{s^2}$
  - Steady state output

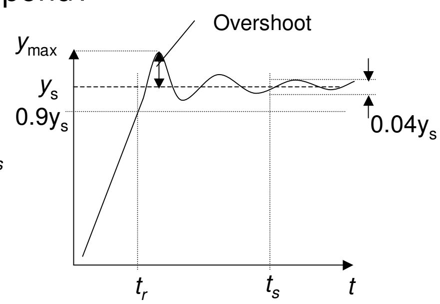
$$y_s=\alpha\frac{a_0b_1-b_0a_1}{a_0^2}+\alpha t\frac{b_0}{a_0}$$



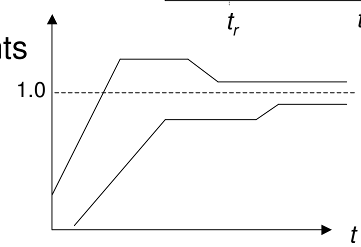
  - Velocity error $e_v(t)=\left|\frac{a_0-b_0}{a_0}t-\frac{a_0b_1-b_0a_1}{a_0^2}\right|$
  - To ensure *finite* velocity error, must have a_0 and b_0 equal
  - To ensure *zero* velocity error, must have a_1 and b_1 equal

Transient Response – speed of response

- How quickly does system respond? Does it overshoot?
- Response characteristics:
  - Rise time (to 90%) = t_r
  - Settling time (until 2% of y_s) = t_s
  - Overshoot $\frac{y_{max} - y_s}{y_s}.100\%$

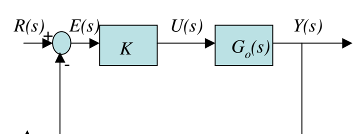


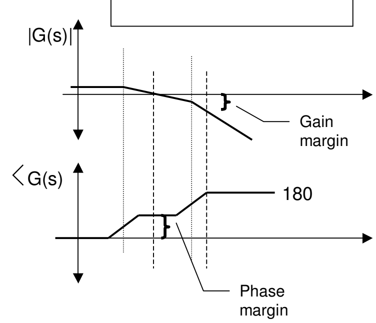
- Set design requirements



Bode – gain and phase margins

- Consider the system with open-loop transfer function $G_o(s)$
  - Gain and phase shift is a function of frequency
- What happens when phase = 180º?
  - E(s) = R(s) – (–) Y(s)
  - Instability!! (unless |Y(s)| very small)
  - Want phase << 180, or gain << 1
- Phase margin is 180-<Y(s) when 20log|G(s)| = 0
- Gain margin is -20log|G(s)| when <Y(s) = 180





Summary

- Accuracy – what needs to be followed?
  - Position, velocity, acceleration
- Pole assignment – closed loop response simply related to poles and zeros of plant and controller
- Stability
  - how close to RHP are the poles?
  - How big are the gain and phase margins?

- Goodwin, Graebe, Salgado: Chapter 7 (pole placement), chapter 8, 9 (design limits)