ENGSCI 332
Control Systems

Lecture 3
Feedback Systems

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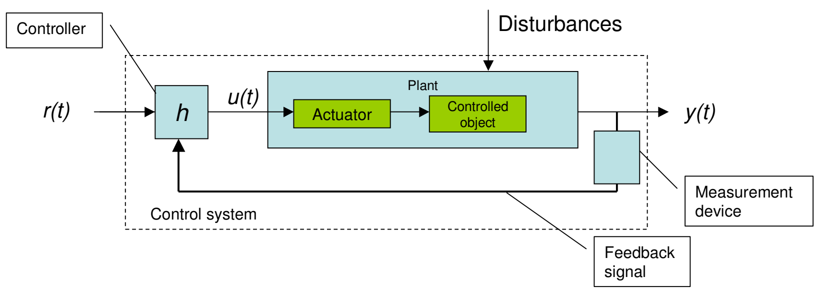
Outline

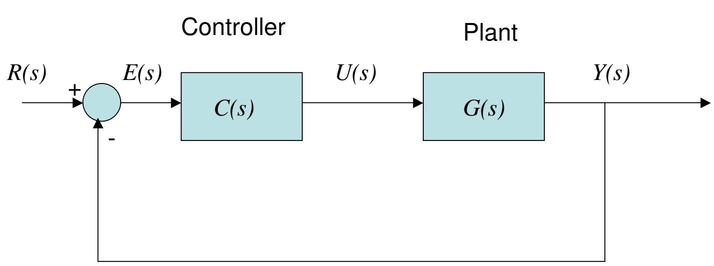
- Concept of feedback
- Classical feedback – PID controller
- Open-loop and closed-loop transfer functions
- Poles and zeros of transfer function

Closed-loop control



- By *measuring* the plant output, can automatically adjust actuator signal so that desired reference is matched
- Actuator signal then depends on both input reference and *feedback* of actual output
  $u(t) = h(\ r(t), y(t)\ )$
  - $u(t)$ determines how well $y(t)$ matches or tracks $r(t)$.

Basic feedback control



$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)}$$ - Closed loop gain

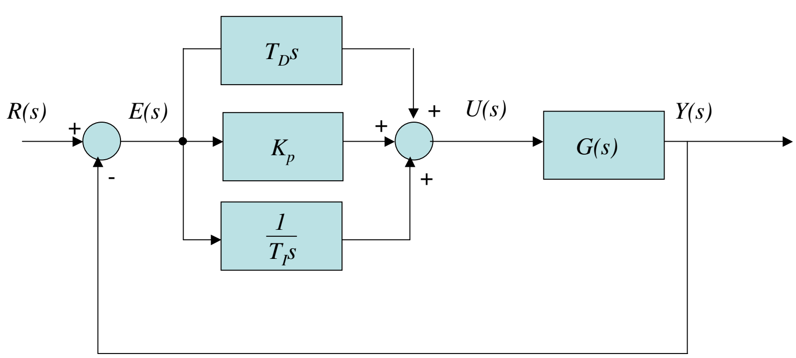
$$\frac{Y(s)}{E(s)} = C(s)G(s)$$ - Open loop gain

Controller design

- Design goal is for Y(s)/R(s) to approach unity
  - i.e. output exactly tracks input
- In open loop, want C(s) to equal inverse of G(s)
  - Ideal situation, doesn't take into account effects of noise
- Note form of closed-loop gain: If C(s)G(s)>>1, then feedback loop in effect cancels out the open-loop transfer function G(s).

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)}$$

PID feedback control



$$\frac{U(s)}{E(s)} = K_P + \frac{1}{T_I s} + T_D s$$
$$= \frac{K(s-z_1)(s-z_2)}{s}$$

- In effect, PID controller can cancel out poles of second order plant *G(s)*

Note: In practice, differentiator has too much gain at high frequencies, so modify: $\frac{T_D s}{1+\tau_D s}$

Typically, $\tau_D \approx 0.2 T_D$

Classical PID feedback

- Determine prediction error $e(t) = r(t)-y(t)$
- Proportional control $u(t) = k_p e(t)$
  - Steady-state error proportional to 1/gain
- Integral control $u(t) = k_i \int e(\tau) d\tau$
  - Accumulate error
  - Response reduces error over time
- Derivative control $u(t) = k_i \frac{d}{dt} e(t)$
  - Increase response to rapid change ("predictive" control responds to increasing error)

PID design

- Ad hoc design rules for determining P, I, and D gains
- E.g. "Reaction curve"
  - Measure open loop step response (i.e. reflecting behaviour of plant)
  - Estimate speed of response (ie initial delay and slope of step response)
  - Set PID gains according to empirical formulae
- "Oscillation method"
  - Gradually increase Proportional gain until system oscillates
  - Measure period of oscillation
  - Set PID gains according to empirical formulae

Summary

- Feedback system allows actual output of system to modify system operation
  - Error between actual output and desired output (reference input) used to “control” the plant
- Feedback alters the overall transfer function between reference input and actual output
- PID controller – proportional, integral, and derivative of error signal

- Goodwin, Graebe, Salgado: Chapter 2 (feedback), chapter 6 (PID control)